# IONIZATION OF AIR BY A PULSED SOURCE OF 

## GAMMA RADIATION

Yu. A. Medvedev, B. M. Stepanov,
UDC 538.561
G. V. Fedorovich, and L. P. Feoktistov

The electromagnetic field generated through the irradiation of air by a pulse of $\gamma$ radiation in the absence of external fields [1-3] and with perturbation by an external magnetic field source [4-7] has not been studied theoretically in great detail in published reports. An experimentally recorded electric field pulse excited by such a source, published by American authors [9], was interpreted in [8] on the basis of the solution of these problems. In the interpretation of data obtained under actual conditions one must allow for the effects of the disturbance of the geoelectric field through the ionization of air in the vicinity of the source. The fact that such effects can be important is indicated by the experimental results presented in [10].

A number of circumstances compel one to assume that these results are connected with the disturbance of the geoelectric field. One of the most important is the difficulty interpreting the results on the basis of the model proposed in [10], where the observed variations of the geoelectric field are connected with the formation of an electric dipole moment $M$ in the vicinity of the source and with the subsequent increase in the height of this moment through floating up in the atmosphere. This is contradicted by the necessity (within the framework of the interpretation proposed in [10]) of assuming that the observed moment M grows by more than 2.5 times during a time when the intensity of all the processes accompanying the phenomenon, including those leading to the formation of the moment, should die out. We note also that the calculated value of the initial moment (in the model of the phenomenon adopted in [10]) is about an order of magnitude less than the observed value and displays a different tendency of the variation with the height $h$ of the source from that obscrved. The explanation proposed below for the observed variations as a result of a disturbance of the geoelectric field describes the experimental data more naturally and does not encounter the cited difficultics inherent to the model of the phenomenon of [10].

Besides the necessity of interpreting the results of observations of slow variations of the type presented in [10], the effects of the disturbance of the geoelectric field must also be taken into account in the interpretation of microsecond pulses of the type presented in [9], since these effects can lead to a quite noticeable contribution to the total signal. In fact, if the initial field $\mathrm{E}^{0}$ is considerably weakened in a volume of size r under the action of a source of $\gamma$ radiation, then by equating the energy of the initial electrostatic field to the energy of the radiation pulse we find that at a distance $s$ from the source a radiation pulse with a characteristic duration $\tau$ will have an amplitude

$$
E \approx E^{0,2 / 2}(c \tau)^{-1 / 2} \mathrm{~s}^{-1}
$$

For a geoelectric field $E^{0} \approx 200 \mathrm{~V} / \mathrm{m}$ with $\mathrm{r} \approx \mathrm{c} \tau \approx 1 \mathrm{~km}$ and $\mathrm{s} \approx 10^{2} \mathrm{~km}$ we have $\mathrm{E} \approx 2 \mathrm{~V} / \mathrm{m}$. This estimate indicates the measurable contribution of the effects of the disturbance of the geoelectric field to the total signal and the necessity of a more detailed study of this phenomenon.
81. Before proceeding to the analysis of the effects of the disturbance of the geoelectric field by a pulsed source of $\gamma$ radiation, let us briefly describe those characteristics of the medium and the geoelectric field in the ground layer of the atmosphere which significantly determine the choice of the model analyzed below. It is known [11] that a vertical electric field (its sign corresponds to a negative charge of the earth's surface) with a magnitude of $50-180 \mathrm{~V} / \mathrm{m}$ exists only at heights less than $h_{0} \approx 2-3 \mathrm{~km}$. At greater heights this field declines rapidly (by about an order of magnitude in the height range of $2-2.5 \mathrm{~km}$ ), which is connected with the presence at this height of a layer of positive charges with a density of $\sim 20$ electron charges per cubic centimeter; the thickness of the layer is $\Delta \approx 0.5 \mathrm{~km}$. The conductivity $\sigma$ of the air changes sharply at the same height $h_{0}$ : whereas at $h<h_{0}$ the conductivity is constant and comprises $\sigma \approx 2 \cdot 10^{-4}$ cgse, at greater heights the conductivity

[^0]grows rapidly, so that $\sigma \approx 10^{-3} \mathrm{cgse}$ already at a height of $\sim 3 \mathrm{~km}$. Currents flow through the conducting air in the layer of $h<h_{0}$, discharging the positive charges in the layer at the height $h_{0}$. The density $\mathbb{E}^{0} \sigma_{0} \approx 10^{-6}$ cgse of these currents is approximately equal to the average curront density over the earth's surface of the lightning discharges which, according to existing concepts [12], "recharge" the layer at the height $h_{0}$.

If under these conditions a pulsed isotropic source of $\gamma$ radiation is turned on at the time $t=0$ at a height $h<h_{0}$, then a spherical front of $\gamma$ radiation, behind which an air conductivity $\sigma(r, t-r / c)$ develops, propagates from it through space with the velocity of light $c$. The conducting volume is polarized in the external field and a transient electric field develops within it. In analyzing the electrodynamic phenomenon at $0<h<h_{0}$ up to times $t_{1}=\min \left\{h / c ;\left(h_{0}-h\right) / c\right\}$ one can take the external modium and the geoelectric field as uniform. At later times this approximation is incorrect. For example, at $t>t_{2}=\max \left\{h / c ;\left(h_{0}-h\right) / c\right\}$ the conducting volume "short circuits" the layer of increased charge concentration at the height $h_{0}$ to the ground, which leads to a decrease in the charge density in the part of the layer located above the source. At later times excess charges start to flow into this region from neighboring regions and a wave of compensating currents spreads out from the center, causing a relatively slow change in the geoelectric field at distances greater than the size of the initial region of decreased charge density in the layer at the height $h_{0}$. It is obvious that at $t>t_{2}$ the effects are essentially connected with the nonuniformity of the medium and of the geoelectric field.

From what has been said it follows that the process of disturbance of the geoelectric field by a pulsed source of $\gamma$ radiation can be divided into three stages, each of which is characterized by its time scale and (as a consequence) by its properties of the occurrence of the phenomenon. The first of these stages is the expansion with the velocity of light of the sphere within which conduction in the air is initiated. In this stage, which lasts up to $10 \mu \mathrm{sec}$ and is characterized by time scales of $\sim 0.1 \mu \mathrm{sec}$, the displacement currents and the excitation of an induced magnetic field are important. Therefore, the most intense emission of wave signals occurs in this stage. In studying the phenomena at these times one can neglect the nonuniformity of the medium and of the geoelectric field.

In the second stage (with the flow of charges down from the layer at the height $h_{0}$ to the ground through the region of increased conductivity), which lasts as long as the source of quanta acts (up to a few seconds), the conduction currents flowing in the volume near the source are important.

At times on the order of tens of seconds and minutes one can assume that the conductivity of the air near the source has decreased so much that the currents near it become considerably smaller than the currents connected with the compensation for the charge of part of the layer at the height $h_{0}$ which has flowed to the ground by charges from neighboring regions. Here the conduction currents flowing through the layer at the height $h_{0}$ become important. The inhomogeneity of the medium and of the geoelectric field are important in the last two stages. The relative slowness of the processes makes it possible to neglect the effects of induction of a magnetic field by the currents and effects of the emission of wave signals.

Let us proceed to a more detailed study of the process in the stages described.
82. The polarization of the conducting region expanding with the velocity of light leads to the appearance in it of a transient electromagnetic field with components $\mathrm{E}_{\mathrm{r}}(\mathrm{r}, \vartheta, t), \mathrm{E}_{\vartheta}(\mathrm{r}, \vartheta, \mathrm{t})$, and $\mathrm{H}_{\varphi}(\mathrm{r}, \vartheta, \mathrm{t})$. The pulse of $\gamma$ radiation simultaneously excites a wave of curronts of Compton electrons in the air, leading to the excitation of the same field components (see [1-3], for example), but by virtue of the linearity of the problem the effects connected with the initial field and with the side curronts can be analyzed separately. A specific feature of this problem, causing difficulties in its solution, consists in the fact that the radiative currents are not given and must be found together with the fields from the system of Maxwell equations.

The space-time distribution of the field disturbed by the conductivity $\sigma(r, t-r / c)$ developing under the action of the $\gamma$ radiation satisfies the Maxwell equations

$$
\begin{equation*}
\operatorname{rot} \mathrm{H}=\frac{1}{c} \frac{\partial \mathrm{E}}{\partial t}-\frac{4 \pi}{c} \sigma \mathrm{E}, \quad \operatorname{rot} \mathrm{E}=-\frac{1}{c} \frac{\partial \mathrm{H}}{\partial t} \tag{2.1}
\end{equation*}
$$

for the field components $\mathrm{E}_{\vartheta}, \mathrm{E}_{\mathbf{r}}$, and $\mathrm{H}_{\varphi}$ with the initial conditions

$$
E_{\vartheta}(r, \vartheta, 0)=E_{\vartheta}^{0}(r, \vartheta), \quad E_{r}(r, \vartheta, 0)=E_{r}^{0}(r, \vartheta), \quad I I_{\varphi}(r, \vartheta, 0)=0 .
$$

As the boundary conditions we require that the condition $\mathrm{E}_{\vartheta}(a, \vartheta, t)=0$ be satisfied at the surface of a sphere of sufficiently small radius $a$ (we assume that the source is surrounded by a perfectly conducting sphere of radius $a$ ) and that $\mathrm{E}_{\vartheta(\mathrm{r}, \vartheta, t)-\mathrm{E}_{\vartheta}(\mathrm{r}, \vartheta)=\mathrm{H}_{\varphi}(\mathrm{r}, \vartheta, \mathrm{t}) \text { at large enough distances outside the conduction zone. }}^{\text {z }}$ As in the problem considered in [4], these conditions determine the unique solution of the problem. With the given initial conditions the Maxwell equations contain the independent variables $r, v$, and $t$. The angular
dependences are separated in the equations and the boundary conditions if the field distributions (including the initial ones), as in [3], are represented in the form

$$
\begin{gather*}
E_{r}(r, \vartheta, t)=\sum_{i} E_{r l}(r, t) \cdot P_{l}(\cos \theta) . E_{\vartheta}(r, \vartheta, t)=\sum_{l} \frac{1}{r} E_{\theta l}(r, t) \cdot P_{1}^{(1)}(\cos \theta)  \tag{2.2}\\
H_{\varphi}(r, \vartheta, i)=\sum_{i} \frac{1}{r} H_{\varphi l}(r, t) \cdot P_{1}^{(1)}(\cos \vartheta)
\end{gather*}
$$

where $P_{l}(x)$ are polynomials while $P_{l}^{(1)}(x)=\left(1-x^{2}\right)^{1 / 2}\left[d P_{l}(x) / d x\right]$ are associated Legendre polynomials. The coefficients of the expansions of the fields by polynomials satisfy the one-dimensional nonsteady equations separately for each $l$ :

$$
\begin{gathered}
\frac{1}{c} \frac{\partial E_{r l}}{\partial t}=\frac{l(l+1)}{r^{2}} \Pi_{\Phi i}-\frac{4 \pi}{c} \sigma E_{r l}, \quad \frac{1}{c} \frac{\partial E_{0 l}}{\partial t}=-\frac{\partial H_{\Phi i}}{\partial r}-\frac{4 \pi}{c} \sigma E_{\hat{}} \\
\frac{1}{c} \frac{\partial H_{i t}}{\partial t}=-\frac{\partial I_{\Phi l}}{\partial r}-E_{r i} .
\end{gathered}
$$

The use of expansions of the angular dependences (2.2) for the fields allows one to separate the angular dependence also in the problem of the disturbance of the initial field by a source located at the boundary between the air and a perfectly conducting half-space. Since the effect also occurs in the case of a uniform space, the effect of the additional asymmetry introduced by the underlying surface on the characteristics of the disturbed fields is not taken into account below.

Let us consider the problem of the disturbance of the initial electric field which is described by the solution of the static problem of a conducting sphere of radius $a$ in a uniform external field:

$$
\begin{gathered}
E_{\vartheta}(r, \vartheta, 0)=E_{0}\left(1-a^{3} / r^{3}\right) \sin \vartheta \\
E_{r}(r, \vartheta, 0)=-E\left(1-2 a^{3} / r^{3}\right) \cos \vartheta, r \geqslant a .
\end{gathered}
$$

At first, as in [3] we will only allow for the electronic conductivity of the air (it predominates over the ionic conductivity for a time on the order of microseconds from the start of the irradiation of the air)

$$
\sigma(r, t)=e h \mu^{3} v N e^{-x} x^{-9} r(y-x) / 4 \pi
$$

where the dimensionless function $r(y)$ is found from the equation

$$
\begin{gathered}
d r^{\prime} d y+(\gamma / \mu c) r=\varphi(y), r(0)=0 \\
\varphi(y)=y \mathrm{e}^{\Omega y} / I \cdot\left[A+\mathrm{e}^{\left(\Omega+\Delta_{1}\right) y}\right], I=\int_{0}^{\infty} d y y \mathrm{e}^{\Omega y} /\left[A+\mathrm{e}^{\left(\Omega+\Delta_{1}\right) y}\right] \\
y=\mu c t, x=\mu r
\end{gathered}
$$

(this and the remaining notation coincide with the notation of [3]). We choose the value of the initial field $\mathrm{E}^{0}$ as the scale of the field, and then the expansions (2.2) take the form

$$
\begin{aligned}
E_{r}(x, y, \vartheta)= & E^{0} E(x, y) \cos \vartheta, E_{\vartheta}(x, y, \vartheta)=E^{0}(1 / x) \varepsilon(x, y) \sin \vartheta \\
& H_{\varphi}(x, y, \vartheta)=E^{0}(1 / x) h(x, y) \sin \vartheta
\end{aligned}
$$

where the dimensionless functions $E, \varepsilon$, and $h$ are defined as the solutions of the system of equations

$$
\begin{gathered}
\partial E / \partial \tau=\left(2 / x^{2}\right) h-\sigma^{\prime}(x, \tau) E, \partial \varepsilon / \partial \tau=-\partial h / \partial x+\partial h / \partial \tau-\sigma^{\prime}(x, \tau) \varepsilon, \\
\partial h / \partial \tau=-\partial \varepsilon / \partial x+\partial \varepsilon / \partial \tau-E
\end{gathered}
$$

with the initial conditions

$$
h(x, 0)=0, \quad \varepsilon(x, 0)=x-x_{0}^{3} / x^{2}, \quad E(x, 0)=-1-2 x_{0}^{3} / x^{3}, \quad x_{0}=\mu a
$$

and the boundary conditions

$$
\begin{array}{r}
\varepsilon\left(x_{0}, \tau\right)=0, \varepsilon\left(x_{1}, \tau\right)-\varepsilon\left(x_{1}, 0\right)=h\left(x_{1}, \tau\right), x_{1} \gg 1, \\
\sigma^{\prime}(x, \tau)=4 \pi / \mu c \cdot \sigma(x, \tau)=R x^{-2} e^{-x} r(\tau), R \equiv e k \mu^{2} v N / c .
\end{array}
$$

In solving this problem we used the same values of the physical constants and the same algorithm as in [3].
The results of the numerical integration are presented in Figs. 1-4. The dependences of the field components for $R=1.54 \cdot 10^{5}$ on the "local" time $\tau=(y-x)$ at different distances from the center are shown in Figs. 1-3, while the spatial distributions of the component $E_{\vartheta}$ for the same source at times when the front of the disturbance reaches distances of $5,10,15$, and 20 are shown in Fig. 4.


Fig. 1


Fig. 2

Let us briefly discuss the results obtained. One can see from Figs. 1-3 that at small distances ( $\mathrm{x} \lesssim 2$ ) at the initial times both components of the electric field decline in absolute value to zero (from the numerical calculation at $x=0.4$ for $R=1.54 \cdot 10^{6}$ the damping time of the field is $\sim 0.06$; it decreases with an increase in the intensity of the source) and $E \simeq 0$ and $\varepsilon \simeq 0$ during the time of $\sim 1.5-2$, whereas the magnetic field grows during this time from zero to a value equal to the initial field, and $h \sim x$ so long as $E \approx \varepsilon \simeq 0$. These features of the behavior of the fields in the zone of action of the source were already clear from the properties of the solution of the model problem [13] of the expansion with the velocity of light of an ionization region with a finite conductivity into an external electric field. As shown in [13], in the case of a sufficiently high conductivity the electrical energy of the initial field is transformed into the energy of the magnetic field within the ionization region.

One can ascertain that at distances on the order of several units the surface corresponding to some sufficiently high value of the conductivity moves with a velocity very close to the velocity of light, while at distances $\mathrm{x} \approx 1$, at $\mathrm{R}=1.54 \cdot 10^{5}$, for example, the condition of high conductivity ( $\sigma^{\prime} \tau \gg 1$ ) is satisfied at $\tau \approx 1-2$, so that the conditions of [13] in the given concrete case are satisfied at $\mathrm{x} \lesssim 1-2$ and $\tau \lesssim 1-2$. At larger $\tau$ the condition of strong conduction ceases to be satisfied, the magnetic field begins to die out at small distances, and the components $\mathrm{E}_{\mathrm{r}}$ and $\mathrm{E}_{\vartheta}$ of the induced electric field appear.

The transformation of the shape of the pulse with distance is clearly shown in Fig. 4. It is seen that the first half-wave of the pulse is formed within distances of $x \approx 10$, while $E_{\vartheta} \approx H_{\vartheta} \sim x^{-1}$ at $x>10$.

It is also seen that when the emitted pulse has "departed" to a large distance a disturbed electric field of the dipole type remains at relatively small distances ( $x \approx 1-10$ ). This is a consequence of the fact that in the transient conducting medium in the zone of the source a space charge develops owing to the polarization of the nonuniform conducting volume, and this charge does not vanish as $\sigma \rightarrow 0$, so that in the zone of the source where the conduction was a static electric field distribution remains at large $t$. The spatial distribution of the magnetic field for the same times is close to the distribution of $\mathrm{E}_{\vartheta}$ and $\mathrm{H}_{0}$; in contrast to the electric field, the magnetic field does not remain near the source after the passage of the pulse. This fact is a consequence of the initial model, in which the source dies out with time. In any other model, such as when the intensity of such a source declines not to zero but to a finite value, the residual fields from a transient pulse of $\gamma$ radiation will disappear with time, but a different field distribution, analyzed in [14], arises in the current zone at large $t$.

From Fig. 5 [in which the emitted signal $(x=60)$ is presented for different source intensities: 1 ) $R=$ $7.7 \cdot 10^{2}$; 2) $R=7.7 \cdot 10^{3}$; 3) $R=1.5 \cdot 10^{5}$; 4) $R=1.5 \cdot 10^{6}$; 5) $R=1.5 \cdot 10^{7}$; 6) $\left.R=1.5 \cdot 10^{8}\right]$ it is seen that the amplitude and duration of the emitted signal depend logarithmically on the intensity of the source, which is explained by the logarithmic dependence on N of the effective size of the ionization zone in which the initial field is strongly disturbed.
83. As noted in Sec. 1, the solution presented above describes the phenomenon only up to the time when the external medium and the geoelectric field can be considered as uniform, i.e., actually up to times $\tau \lesssim 3-5$ $\mu \mathrm{sec}$. For the solution of the equations at later times the use of the expansions (2.2) does not simplify the problem, and the angular dependence of the fields must be determined simultaneously with the dependence on distance and time, i.e., the problem becomes multidimensional. The solution (even numerical) of the multidimensional problem presents considerably greater difficulties than the solution of the one-dimensional problem presented in Sec. 2, but the character of the phenomenon at later times can be clucidated on the basis of rather simple estimates.


Fig. 3


Fig. 4

Let us assume that under the conditions described in Sec. 1 corresponding to the lower atmosphere, a source of $\gamma$ radiation is turned on at a height $h<h_{0}$ with a high enough activity that the conductivity of the air between the layer of positive charges and the earth's surface grows considerably. This leads to the rapid flow of charges down from the part of the layer located above the source to the ground through the region of increased conductivity. After the formation of a region with a reduced concentration of charges in the layer at a height $h_{0}$ excess charges begin to flow into it from neighboring regions and a wave of compensating currents travels out from the center, causing a change in the geoelectric field at large distances. Such a separation of the process into two stages is possible when the difference between the conductivity near the source and the natural conductivity of the air is large enough, and therefore in analyzing the first stage of the process the flow of charges to the ground) one can neglect the effects of charge compensation in the layer due to the current from the neighboring regions, while in analyzing the second stage of the process (waves of compensating currents) one can neglect the change in the natural conductivity of the air, assuming that at the times of interest in this case (on the order of seconds and minutes) the additional ionization of air by the source has ceased.

Let us estimate the size of the region whose charge is carried to the ground by the current $\sigma \mathrm{E}$. The value of the field E can be estimated from the Maxwell equations. In this case at times $\mathrm{t} \sim \mathrm{h}_{0} / \mathrm{c}$, when the zone of increased conductivity reaches sizes of $\sim h_{0}$, one can ignore the effect of the magnetic field at distances of $\sim \mathrm{h}_{0}$ from the source; this is seen from the results presented in Sec. 2. In this case from the first equation of the system (2.1) we get

$$
d E / d t \approx-4 \pi \sigma E
$$

from which we obtain the estimate for the field E :

$$
E \approx E^{0} \exp \left(-4 \pi \int_{0}^{+} \sigma d t^{\prime}\right) .
$$

The substitution of $\mathrm{E}^{0}$ in this solution is admissible since the distortions of the external field caused by a brief pulse of $\gamma$ radiation are so small, like the additional magnetic field (see Sec. 2). Since the value of the induced conductivity $\sigma$ depends on the distance $r$ from the source,

$$
r=\sqrt{R^{2}+\left(h_{0}-h\right)^{2}},
$$

where $R$ is the distance from the axis of symmetry, the density of the current from the layer of positive charges is

$$
j(R, t)=E^{0} \sigma(r(R), t) \exp \left\{-4 \pi \int_{0}^{+} \sigma\left(r(R), t^{\prime}\right) d t^{\prime}\right\},
$$

so that the total change $\delta \Sigma(\mathrm{R})$ in the surface charge density in the layer is described by the expression

$$
\begin{equation*}
\delta \boldsymbol{\Sigma}(R)=\int_{0}^{\infty} d t j(R, t) \approx \Sigma^{0}\left\{1-\exp \left(-4 \pi \int_{0}^{\infty} \sigma d t\right)\right\}, \tag{3.1}
\end{equation*}
$$

where $\Sigma^{0}=\mathrm{E}^{0} / 4 \pi$ is the initial surface charge density. At small distances R the exponent in (3.1) can be large (when the activity of the source is high enough) and $\delta \Sigma \approx \Sigma^{0}$, i.e., the charge moves entirely from the layer to the ground. At large distances $R$ the exponent is small and $\delta \Sigma \ll \Sigma^{0}$, i.e., the charge changes little. The boundary $R_{0}$ of the region from which the charge has gone almost entirely to the ground is determined by the condition

$$
\begin{equation*}
4 \pi \int_{0}^{\infty} \sigma\left(r\left(R_{0}\right), t\right) d t=1 \tag{3.2}
\end{equation*}
$$



Fig. 5


Fig. 6
i.e., it depends both on the activity of the source and on its height.

At large enough distances ( $\mathbf{R} \gg \mathbf{R}_{0}$ ) the disturbance of the electric field $E$ connected with the "discharge" of the layer is equivalent to the field of an electric dipole of magnitude

$$
M=2 \pi R_{0}^{2} \Sigma^{0} h_{0}=\frac{1}{2} E^{0} R_{0}^{2} \pi
$$

with the negative charge in the upper part.
Let us consider the dynamics of the motion of compensating charges at times on the order of several seconds and later. Leaving aside the details of the height distribution of the charges, one can consider them as concentrated in some plane lying at a height $h_{0}$; the surface density of the charges will be designated as $\Sigma$. The equilibrium state of the system of charges corresponds to a constant value $\Sigma=\Sigma^{0}$ over the entire plane.

At the initial time the equilibrium state is disturbed, since the charge is entirely removed from the region within a circle of radius $R_{0}$. Instead of the motion of the compensating positive charges one can consider the "spreading" of some equivalent negative charge concentrated in a circle of radius $R *(t)$ and originally distributed with a surface density $\Sigma^{0}$ in a circle of radius $R_{0}$. The density of the equivalent charge varies by the Law

$$
\begin{equation*}
\partial \Sigma / \partial t=(\sigma \Delta / R) \cdot \partial\left[R E_{R}\right] / \partial R, \tag{3.3}
\end{equation*}
$$

where $\mathrm{E}_{\mathrm{R}}$ is the electric field component tangential to the plane. For qualitative estimates one can assume that a uniform charge distribution within the circle $R^{*}(t)$ is retained during the entire time of outflow. In the process, the total charge $\pi \Sigma\left[R^{*}(t)\right]^{2}=\pi \Sigma^{0} R_{0}^{2}$ is conserved, while the electric field $E_{R}$ varies by the law

$$
E_{R}=\left\{\begin{array}{lll}
-\pi \sum R / R^{*} & \text { at } & R<R^{*}  \tag{3.4}\\
-\pi \sum\left(R^{*} / R\right)^{2} & \text { at } & R>R^{*}
\end{array}\right.
$$

Substituting (3.4) into (3.3), we find that at $R<R^{*}$ we have

$$
\partial \underline{S} \partial t=-2 \pi \sigma \Delta \cdot \Gamma / F^{*}(t),
$$

which makes it possible, using the conservation law, to write the solution in the form

$$
\Sigma\left(n<R^{*}, t\right)=\Sigma^{0}\left(R_{0} / R^{*}\right)^{2}, R^{*}=R_{0}+\pi \sigma \Delta t
$$

Thus, the outflow of the effective charge occurs with a constant velocity $\mathrm{D}=\pi \sigma \Delta$ and its density at $t \gg$ $\mathrm{R}_{0} / \pi \Delta \sigma$ declines as $\mathrm{t}^{-2}$.

The dynamics of the compensating currents described leads to a nonmonotonic time dependence of the vertical electric field at a distance $R$ from the axis of symmetry. Actually, at the initial times following the abrupt decline in the field due to the rapid "discharge" of part of the surface charge to the ground the field continues to decline owing to the approach of the "edge" of the region of reduced charge concentration to the detection point. Only after $R^{*}$ eventually becomes greater than $R$ does the field start to decline in proportion to the decline in charge density. A valid expression for the vertical field $E_{V}$ at the earth's surface is

$$
E_{\mathbf{V}}(R)=2 h_{0} \int_{0}^{\infty} R^{\prime} \Sigma\left(R^{\prime}, t\right) d R^{\prime} \int_{0}^{\pi} d \varphi\left[h_{0}^{2}+R^{2}+R^{\prime 2}-2 R R^{\prime} \cos \varphi\right]^{-3 / 2}
$$

which, for a uniform charge distribution in a circle of radius $R^{*}(t)$, has the form

$$
\begin{equation*}
E_{\mathrm{V}}(R)=E^{0} \frac{h_{0}}{\pi}\left(\frac{R_{0}}{R^{*}}\right)^{2} \int_{0}^{\pi} \frac{d \varphi}{h_{0}^{2}+R^{2} \sin ^{2} \varphi}\left[V \overline{h_{0}^{2}+R^{2}}-\frac{h_{0}^{2}+R^{2} \cdots R R^{*} \cos \varphi}{\sqrt{h_{0}^{2}+R^{2}+R^{* 2}-2 R R^{*} \cos \varphi}}\right] . \tag{3.5}
\end{equation*}
$$

In accordance with (3.5) the time dependence of the field is connected with the variation of $R^{*}=R^{*}$ ( $t$ ). The dependence of $E_{V} / E_{0}$ on $R^{*}$ is presented in Fig. 6 for two distances $R=7.8$ and 14.3 km (curves 1 and 2 , respectively; we take $h_{0}=3 \mathrm{~km}$ and $R_{0}=7.5 \mathrm{~km}$ ). It is seen that the maximum value of the field is reached the later, the farther away the detection point is located. The magnitude of the maximum decreases with distance.
§4. Let us compare the results obtained with the available experimental data. First we consider the short-period disturbances of the geoelectric field analyzed in Sec. 2. It is seen from Fig. 5 that for $R=1.54$ $10^{5}$ the amplitude of the signal at a distance $\mathrm{x}=40$ comprises $\sim 10^{-2} \mathrm{E}^{0}$. Taking $\mathrm{E}^{0}$ as equal to the average value of the geoelectric field at zero height, $\mathrm{E}^{0} \simeq 180 \mathrm{~V} / \mathrm{m}[11]$, in accordance with the qualitative estimate we find that the magnitude of the signal at this distance is $\sim 2 \mathrm{~V} / \mathrm{m}$, which, though markedly lower than the observed level of the signal generated by a short pulse of $\gamma$ radiation (according to the results of [9], the amplitude of this signal is $\sim 15$ times higher), is still quite measurable. The effects of a disturbance of the geoelectric field can become controlling when the magnitude of the geoelectric field grows by hundreds or thousands of times during rainfalls, gales, and snowstorms. In this case a change in the polarity of the emitted pulse is possible together with a change in the sign of the geoelectric field.

Let us compare the results of the approximate analysis of the slowly varying fields, due to the flow of positive charges in the layer at a height $h_{0}$ into the region of reduced concentration, with the results of experimental observations of slow variations of the geoelectric field presented in [10]. First of all, we determine the size $R_{0}$ of the region of reduced concentration given by Eq. (3.2). To calculate the integral we use the data of [10] on the source. Using Eqs. (8) and (10) of [10], we write the integral of (3.2) in the form

$$
\begin{equation*}
4 \pi \int_{0}^{\infty} \sigma d t=\frac{4 e \mu_{2}}{r} \sqrt{\frac{\pi N v}{\beta \lambda}} \exp \left[-\frac{1}{2} \int_{0}^{r} \lambda^{-1} d r\right] \int_{0}^{\infty} f^{1 / 2}(t) d t \tag{4.1}
\end{equation*}
$$

(here and below the designations and values of the quantities are the same as those in [10]). As for the normalized function $f(t)$, it is known that it varies as $t^{-1.2}$. To make normalization possible, we must assume that $f\left(t<t_{1}\right)=0$. Then $f(t)$ can be written in the form $0.2 t_{1}^{0.2} t^{-1.2}$. In order that the integral over $t$ in (4.1) have a finite value, it is necessary to assume that $f\left(t>t_{2}\right)=0$. Then

$$
\int_{0}^{\infty} f^{1 / 2}(t) d t \approx \sqrt{1.25} t_{1}^{0,1} t_{2}^{0,4}
$$

The dependence of the final results on $t_{1}$ and $t_{2}$ is very weak, and for determinacy we take $t_{1}=10^{-6}$ sec and $t_{2}=10^{-2}$ sec below. Changes in $t_{1}$ and $t_{2}$ by one to two orders of magnitude change the result by $2-5 \%$. Substituting the numerical values from [10] into (4.1), we obtain

$$
\begin{equation*}
4 \pi \int_{0}^{\infty} \sigma d t=10^{5} r^{-1} \exp \left(-1.7 \xi_{\xi} r\right) \tag{4.2}
\end{equation*}
$$

where $r$ must be given in kilometers; $\xi$ is the mean density of the air in the layer between $h$ and $h_{0}$ (normalized to the air density at zero height). According to [10] (one must allow for the elevation of the earth's surface above sea level), $0.8 \geq \xi \geq 0.6$ (below we take $\xi=0.7$ ). The function (4.2) is equal to unity at $\mathrm{r}=8 \mathrm{~km}$, which leads to a value of $R_{0}=7.5 \mathrm{~km}$, for example, for $h=0.3 \mathrm{~km}$ and $h_{0}=3 \mathrm{~km}$.

The dipole moment connected with the "discharge" of the layer comprises $\mathrm{M} \approx 0.4 \mathrm{C} \cdot \mathrm{km}$ (taking $\mathrm{E}_{0}=$ $40 \mathrm{~V} / \mathrm{m}$ ), which is comparable with that cited in [10]. Here it should be noted that the value of M in [10] was calculated on the basis of the results of measurements of the field $\mathrm{E}_{\mathrm{V}}$ in the model of the phenomenon adopted in [10], according to which the dipole is formed by the charging source and its reflection in the ground. For a more correct comparison of the theoretical and experimental results, one must turn directly to the experimental data presented in [10].

The complicated time dependence of the field at a distance of 7.8 km and the simpler dependence at a distance of 14.3 km attract attention. At a distance of 7.8 km an initial narrow peak (with a duration of $<10$ sec) was recorded in which the disturbance exceeded the initial field, after which there followed a flatter signal, also with a maximum at $t \approx 30 \mathrm{sec}$. The amplitude of the field disturbance at the second maximum is also comparable with the initial field. At a distance of 14.3 km the initial narrow peak is absent, and after the initial jump the disturbance continues to grow, reaching a maximum at $\sim 1.5 \mathrm{~min}$, after which it declines with
a characteristic time of $\sim 5 \mathrm{~min}$. These results can be compared with the results of the calculation of the vertical electric field presented in Sec. 3 if one assumes that $R^{*}$ changed at a rate of $\sim 6 \mathrm{~km} / \mathrm{min}$ from 7.5 km at $t=0$. In this case $R^{*}$ reaches values of $\sim 9 \mathrm{~km}$, at which the field disturbance at a distance of 7.8 km is greatest, at a time of $\sim 15 \mathrm{sec}$, which coincides with the observed value.

Values of 17 km , at which the field disturbance at a distance of 14.3 km is greatest, are reached by $\mathrm{R}^{*}$ at $t \approx 1.5 \mathrm{~min}$, which also agrees with the observed time. The calculated amplitudes of the field disturbances are comparable with the observed amplitudes (the differences do not exceed twofold).

The initial narrow peak observed at a distance of 7.8 km is not explained within the framework of the model of Sec. 3. Its nature is evidently connected with an increase in the conductivity of the air in the immediate vicinity of the detection point under the action of the $\gamma$ radiation. The local character of such an effect (it is absent at a distance of 14.3 km ) supports this point of view.

It should be noted that to provide the observed velocity of the wave of compensating currents one must take a conductivity of $\sim 5 \cdot 10^{-2} \mathrm{sec}^{-1}$ in the layer were these currents flow, which exceeds the natural conductivity at these heights ( $\sim 3-4 \mathrm{~km}$ ) by about an order of magnitude. Such values of the conductivity are evidently connected with the irradiation of the air. To refine the model of Sec .3 one would have to consider the three-dimensional motion of the wave of compensating currents in a medium with a conductivity which varies with time and with distance, which takes the problem beyond the limits of simple estimates, however.

The authors thank G. G. Vilenskaya, who performed the numerical calculations.

## LITERATURE CITED

1. A. S. Kompaneets, "The radio emission of an atomic explosion," Zh. Eksp. Teor. Fiz., 35, No. 6 (12) (1958).
2. V. Cilinsky, "Kompaneets model for radio emission from a nuclear explosion," Phys. Rev. A, 137, No. 1 (1965).
3. G. G. Vilenskaya, V. S. Imshennik, Yu. A. Medvedev, B. M. Stepanov, and L. P. Feoktistov, "The electromagnetic field excited in air by a transient source of gamma radiation located on a perfectly conducting plane," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1975).
4. W. J. Karzas and R. Latter, "The electromagnetic signal due to the interaction of nuclear explosions with the earth's magnetic field," J. Geophys. Res., 67, 4655 (1962).
5. Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "Rapid disturbances of the geomagnetic field by a transient source of gamma quanta," Geomagn. Aeron., No. 1, 281 (1971).
6. Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "The radio emission accompanying a disturbance of the geomagnetic field by a transient source of gamma radiation," Geomagn. Aeron., No. 2, 191 (1972).
7. G. G. Vilenskaya, Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "Disturbance of a magnetic field by a transient source of gamma quanta," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1975).
8. V. V. Ivanov, Yu. A. Medvedev, B. M. Stepanov, and G. V. Fedorovich, "On the nature of the signal emitted from a total electromagnetic pulse," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1977).
9. J. K. Johler and J. C. Morgenstern, "Propagation of the ground wave electromagnetic signal with particular reference to a pulse of nuclear origin," Proc. Inst. Electr. Eng., No. 53, 2043 (1965).
10. R. E. Holzer, "Atmospheric electrical effects of nuclear explosions," J. Geophys. Res., 77, No. 30, 5845 (1972).
11. Handbook on Geophysics [in Russian], Nauka, Moscow (1965).
12. R. P. Feynman, Lectures on Physics, Vol. 5, Addison-Wesley (1963-1965).
13. V. K. Bodulinskii, Yu. A. Medvedev, and G. V. Fedorovich, "Disturbance of external fields by conducting regions expanding with the velocity of light," Radiotekh. Elektron., 17, No. 2, 283 (1972).
14. Yu. V. Gutop and Yu. A. Medvedev, "On the electromagnetic field generated through the ionization of air by a point source of gamma radiation in the electric field of the earth," Radiofizika, 12, No. 10, 1573 (1969).

[^0]:    Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 14-24, March-April, 1978. Original article submitted February 14, 1977.

